SEMINAR: GEOMETRY OF TEICHMÜLLER SPACE

The Teichmüller space of a closed surface S of genus $g \geq 2$ can be described in several ways. On the one hand, it is the space of equivalence classes of marked hyperbolic structures on S where two metrics are equivalent if there are isometric with an isometry which is homotopic to the identity. It can also be described as the space of equivalence classes of discrete faithful representations of the fundamental group $\pi_1(S)$ of the surface into the group $PSL(2, \mathbb{R})$ of orientation preserving isometries of the hyperbolic plane, where two such representations are equivalent if they are conjugate. Furthermore, it is a space of equivalence classes of marked complex structures on S.

The goal of this seminar is to introduce several geometric aspects of Teichmüller space. We begin with the topological classification of closed surfaces, followed by some basic notions of hyperbolic geometry. We discuss different descriptions of Teichmüller space and introduce natural systems of coordinates, the so-called Fenchel Nielsen coordinates. The mapping class group of the surface S is introduced as well as its action on Teichmüller space. There are two celebrated mapping class group invariant metrics on Teichmüller space which will be discussed as well.

The seminar is mainly intended for master students with some background in geometry. The first part of the seminar only uses elementary topological tools and are suitable for Bachelor students.

Interested students should write an email by Wed. 21. July to *ursula@math.uni-bonn* indicating the following:

- if you are Bachelor or Master student
- it you have background in (Riemannian) geometry
- the talks of the list below you are willing to give - please list several, if possible, to simplify the process
- if you are willing to work with some other student on a topic that covers 2 (or more) talks - also with whom, in case you know

List of talks:

(1)–(3) Topological classification of closed surfaces and their fundamental groups - *Fulton*: Ch. 7.,8. and 17.
(4)–(5) Hyperbolic surfaces - *Buser*: Ch. 1.1.–1.6.
(6)–(7) Fenchel-Nielsen Coordinates - *Buser*: Ch. 1.7., 3.1.–3.6.; see also *Farb-Margalit*: Ch. 10.5.–10.7.

Teichmüller metric:

(8) Measured foliations - *Farb-Margalit*: Ch. 11.2.
(9) Quadratic differentials - *Farb-Margalit*: Ch. 11.3.

(10) Quasi conformal maps and Teichmüller maps - *Farb-Margalit*: Ch. 11.1. and 11.4.
(11) Uniqueness of Teichmüller maps - *Farb-Margalit*: Ch. 11.5. and 11.6.
(12)–(13) Existence of Teichmüller maps and Teichmüller metric - *Farb-Margalit*: Ch. 11.7. and 11.8.

References:

- W. Fulton, *Algebraic Topology - A first course*, Graduate Texts in Mathematics
- P. Buser, *Geometry and Spectra of Compact Riemann Surfaces*, Birkhäuser
- B. Farb and D. Margalit, *A Primer on Mapping Class Groups*, Princeton University Press